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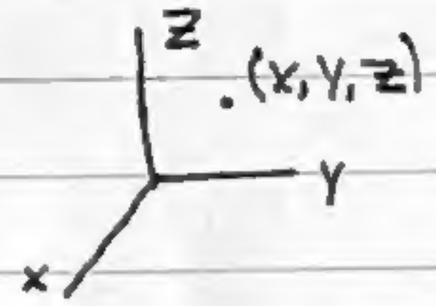
12.1 - Coordinates in 3-space ($\mathbb{R}^3$)

($\mathbb{R}^2$) 2-space: 

$\mathbb{R}^2 = \{(x,y) : x,y \in \mathbb{R}\}$

($\mathbb{R}^3$) 3-space: 

$\mathbb{R}^3 = \{(x,y,z) : x,y,z \in \mathbb{R}\}$

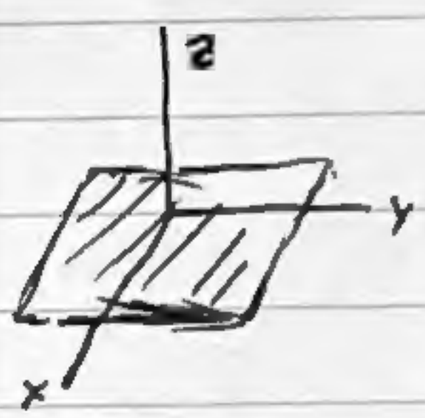
$\mathbb{R}$ coordinates of a point (x,y,z) are x,y,z...

I. Coordinate planes

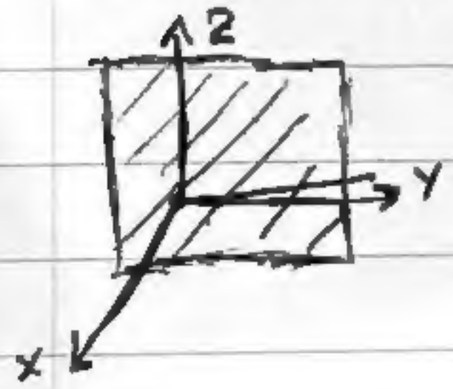
> A coordinate plane in $\mathbb{R}^3$ is a set of points, all of which have a (predetermined) coordinate set to 0.

ex/

$\mathbb{R}$ xy plane in $\mathbb{R}^3$ is $\mathbb{R}$ $z=0$ plane, i.e. $\{(x,y,z) \in \mathbb{R}^3 : z=0\}$



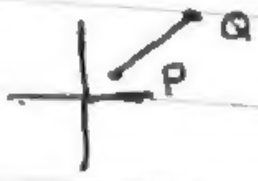
$\mathbb{R}$ yz plane in $\mathbb{R}^3$ is $\mathbb{R}$ $x=0$ plane, i.e. $\{(x,y,z) \in \mathbb{R}^3 : x=0\}$

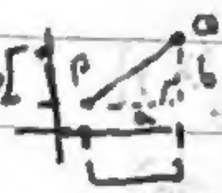


Distances in 3-space

2space

$\mathbb{R}^2$:

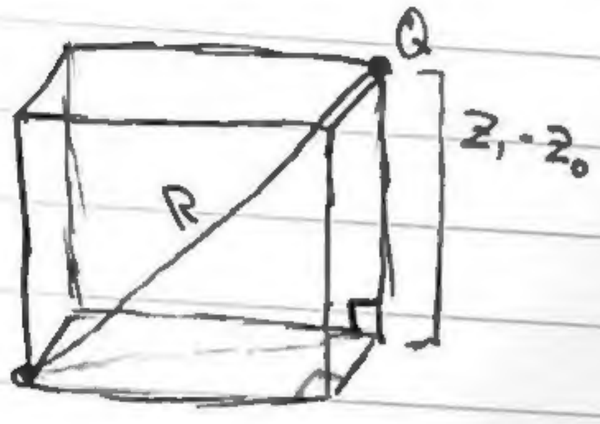




$d(P,Q) = \sqrt{a^2+b^2} = \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2}$

$d(P,Q) = \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2}$

3 Space



$$R = \sqrt{(z_1 - z_0)^2 - \left(\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}\right)^2}$$

$$R = \sqrt{(z_1 - z_0)^2 + (x_1 - x_0)^2 + (y_1 - y_0)^2}$$

Thm: Let $P = (x_0, y_0, z_0)$ and $Q = (x_1, y_1, z_1)$ be points in $\mathbb{R}^3$.
The distance between them is

$$\boxed{d(P,Q) = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}}$$